Outage Probability based Comparison of Underlay and Overlay Spectrum Sharing Techniques

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***Abstract*— Three spectrum sharing schemes - spreading-based underlay, interference avoidance (IA) based overlay and spreading-based underlay that implements IA, are compared in this paper. The comparison is based on the influence of these techniques on the performance of an existing legacy static radio system with which the spectrum-sharing radios coexist. Outage probability is used to evaluate the performance of the static radio system. It is shown in the paper that IA techniques dramatically reduce the interference at the static receiver. Hence the performance of an IA-based overlay scheme is much better than a spreading-based underlay scheme. However, a spreading-based underlay scheme that incorporates IA provides benefits over the IA-based overlay scheme. The advantage obtained is more pronounced when the bandwidth available to the agile radio system is increased. It is also shown that the degradation in performance in the absence of perfect information about the static radio system is lesser for the IA-based spectrum underlay scheme than the IA-based spectrum overlay scheme.**

I. Introduction

The FCC decision to open the TV band for wireless broadband has fueled considerable interest in dynamic spectrum sharing techniques. Two of the main approaches that have been proposed for a spectrum-sharing (SS) radio system that intends to co-exist with a static radio system (the term static radio refers to existing legacy users in the spectrum) are the interference averaging or spreading-based spectrum underlay technique and the interference avoidance (IA) based intelligent spectrum overlay technique. Interference averaging refers to transmission techniques where radios spread their signals across the entire bandwidth available to the spectrum sharing radio system. No single source dominates the interference caused to the static radio system in this scheme. Code Division Multiple Access (CDMA) [1] and Ultra Wide Band (UWB) [2] are some examples of this technique. IA-based spectrum overlay is a cognition-based narrowband (NB) technique where the SS radio dynamically chooses frequency bands for transmission. The choice is made such that the interference caused to the static radio system is minimized. This technique requires knowledge of the locations and transmissions of the static radios which can be obtained by sensing. Some example schemes are dynamic spectrum allocation [3] and dynamic channel assignment ([4], [5] and [6]). An alternate approach is to use an interference-averaging-based underlay technique that also employs IA. In this transmission technique, the SS radios spread their signal over the entire available bandwidth. However, they also avoid frequencies in which they can sense static radio transmissions. IA techniques like notching ([7] and [8]) and waveform adaptation ([9] and [10]) can be used to this end.

The three spectrum overlay techniques described above are compared in this paper based on their impact on the performance of the static radio system. This is done by evaluating the interference at a single static receiver present in a field of interfering SS radios. This provides a measure of the average interference caused to the static radio system. Outage probability is used as the performance metric. The analysis in the paper uses a characteristic function based numerical approach and a Gaussian approximation based approach to model the average sum interference at the static receiver due to the SS radios. This is then used to obtain a distribution for the outage probability at the static receiver.

This paper is organized as follows. In Section II, the system model and spectrum sharing techniques analyzed in this paper are delineated. The outage probability analysis requires the computation of the interference statistic at the static receiver. Two different techniques that can be used to calculate the interference statistics at the static receiver are discussed in Section III. In Section IV, outage probability distributions are derived for the spectrum sharing schemes with perfect, no and imperfect system knowledge. In Section V, the outage at the static radio receiver caused by the spectrum sharing schemes is analyzed and compared for some example scenarios. Finally, Section VI presents conclusions and future research directions.

II. System Model

The analysis presented in this paper, models the SS radio interference to a static receiver as a random variable equal to the sum of transmit powers of the SS radios multiplied by a path loss factor. The analysis ignores the interference caused by other static radios to the static receiver. The SS radios are assumed to be uniformly distributed in a circular area with radius extending to infinity around the static receiver. The number of SS radio transmit nodes in the circular region is assumed to follow a Poisson process with parameter N that denotes the average number of transmit nodes per unit area. Such a distribution for interfering radio nodes has been

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previously considered in [11], [12] and [13]. In [14], the use of the Poisson distribution to model interferers is extensively discussed. The probability that there are k nodes in a region with unit area is given by

$$P(k) = \frac{e^{-N} N^k}{k!}. \quad (1)$$

The static radio system is assumed to be a narrowband (NB) system and B is assumed to be its transmission bandwidth. The data transmission bandwidth of the SS radio is also assumed to be B without loss of generality (The outage probabilities can be appropriately scaled if this is not the case). Let the total bandwidth available for the SS radio system be $N_B B$, for some integer N_B and let the power received from an SS radio transmitter at distance of $1m$ be P_a. Let the minimum SIR required at a static receiver for successful transmission be γ and the distance between a static transmitter and static receiver be r_s. Then an outage is caused at the static receiver if

$$\frac{P_t/r_s^\alpha}{\sum_J P_a/r_i^\alpha} \leq \gamma. \quad (2)$$

Here, J is the set of all SS radio transmit nodes in the system which transmit in the same frequency as the static transmitter, P_t is the power received from the static transmitter at a distance of $1m$ from it, r_i is the distance of the i^{th} SS radio node from the static receiver and α is the path loss exponent. This implies that for a given fixed distance r_s between the static transmitter and receiver, an outage is caused at the static receiver if the interference power is above threshold P_i, i.e. if the following condition is satisfied.

$$\sum_J P_a/r_i^\alpha \geq P_i = \frac{P_t/r_s^\alpha}{\gamma} \quad (3)$$

The performance of the following spectrum sharing schemes for the SS radio system are analyzed and compared in this paper.

A. Scheme1: Spectrum Overlay Scheme with Interference Avoidance (IA)

The SS radio system is assumed to be a NB system in which the radios can sense their environment. A radio transmits in some frequency f only if it does not hear any static radio transmission in this frequency band. In the analysis considered in this paper, the SS radios are assumed to choose a transmission frequency with equal probability from the set of available frequencies in which the radios do not sense static radio transmissions.

B. Scheme2: Spreading-based Spectrum Underlay Scheme

The SS radio system is assumed to be a wideband (WB) system. The radios spread their transmission power over the entire available bandwidth $N_B B$. Hence the transmission power over any transmission band of width B is P_a/N_B.

C. Scheme3: Spreading-based Spectrum Underlay Scheme with Interference Avoidance

The SS radio system is assumed to be a WB system. The radios spread their transmission power over the entire available bandwidth $N_B B$. Hence the transmission power over any transmission band of width B is P_a/N_B. The radios also null or notch frequencies in which they can sense transmissions from the static radio system.

III. Interference Statistics for Outage Probability Analysis

The interference at the static receiver is modeled by a random variable equal to sum of the powers of the signals received from the SS radios. The signals received from the SS radios are assumed to have suffered a loss in power that follows exponential propagation laws. Let $g(r)$ be the power of a unit energy signal at distance r from the transmitter of the signal. $g(r)$ is assumed to satisfy the following properties.

1) $g(r)$ is assumed to be monotonically decreasing.

$$\lim_{r\to 0} g(r) = \infty \quad (4)$$

$$\lim_{r\to \infty} g(r) = 0 \quad (5)$$

2) Path loss exponent α is greater than 2.

$$\lim_{r\to \infty} r^2 g(r) = 0 \quad (6)$$

If Equation 6 is not satisfied, the interference power at a given receiver would be a function of the network size and would be infinite for an infinite sized network ([11] and [15]). Also the characteristic function of the interference power would not exist [14]. For the analysis presented here, $g(r)$ is specified as,

$$g(r) = \frac{1}{r^\alpha}; \quad \alpha > 2 \quad (7)$$

The following two methods are used to calculate the interference statistics at the static receiver. The first is a numerical approach based on the characteristic function of the interference variable ([11], [12], [13] and references within). The second uses a Gaussian approximation with correction terms based on higher order cumulants to model the interference. The second approach is computationally less intensive than the first approach. However, the Gaussian approximation is only valid when the interfering agile radios are not too close to the static receiver.

A. Characteristic Function based Numerical Approach

Assume that the static receiver is at the origin and r_i is the distance between the static receiver and the i^{th} SS radio. Let X_a denote the sum of interference powers from radios which are Poisson distributed in a disc of radius a around the static receiver with parameter N. Then,

$$X_a = \sum_{J_{0,a}} g(r_i) \quad (8)$$

Here, $J_{0,a}$ denotes the set of interfering nodes at distance r_i from the static receiver such that $0 \leq r_i \leq a$. The analysis in

[11] derives the cumulative distribution of interference from a Poisson field of interferers by using the characteristic function of X_a. A sketch of this analysis is given here.

The characteristic function of X_a for a given parameter N is given by

$$\phi_{X_a}(\omega, N) = \mathbf{E}\left(e^{i\omega X_a}\right) \tag{9}$$

This may be evaluated by conditioning on the distribution of the number of radios (Poisson distribution).

$$\begin{aligned}\mathbf{E}\left(e^{i\omega X_a}\right) =& \mathbf{E}\left(\mathbf{E}\left(e^{i\omega X_a}|k\ in\ D_a\right)\right)\\ =& \sum_{k=0}^{\infty} \frac{e^{-N\pi a^2}\left(N\pi a^2\right)^k}{k!}\mathbf{E}\left(e^{i\omega X_a}|k\ in\ D_a\right)\end{aligned} \tag{10}$$

Here, D_a denotes a disc of radius a. If the SS radios are uniformly distributed in radius a, the distribution of the radios at any distance r from the center of the disc is given by

$$f_r(r) = \frac{2r}{a^2}; \quad 0 \leq r \leq a. \tag{11}$$

The characteristic function of a sum of random variables is the product of the individual characteristic functions. Hence, the expectation term in Equation (10) can be replaced as follows.

$$\phi_{X_a}(\omega, N) = \sum_{k=0}^{\infty} \frac{e^{-N\pi a^2}\left(N\pi a^2\right)^k}{k!}\left(\int_0^a \frac{2r}{a^2}e^{i\omega g(r)}dr\right)^k \tag{12}$$

This simplifies to

$$\phi_{X_a}(\omega, N) = \exp\left(N\pi a^2\left(\int_0^a \frac{2r}{a^2}e^{i\omega g(r)}dr - 1\right)\right) \tag{13}$$

Letting $a \rightarrow \infty$ and $g(r) = \frac{1}{r^\alpha}$, the integral can be evaluated as

$$\phi_{X_\infty}(\omega, N) = \exp\left(-N\pi\Gamma(1-\beta)e^{-\pi\beta/2}\omega^\beta\right);\ \omega \geq 0 \tag{14}$$

where, $\beta = \frac{2}{\alpha}$ and $\Gamma(.)$ is the Gamma function.

The probability of outage for a threshold P_i can in general be expressed as [16],

$$\begin{aligned}p_{out}(P_i) &= prob\left(X_\infty > P_i\right)\\ &= 1 - \frac{2}{\pi}\int_0^\infty Re\left(\phi_{X_\infty}(\omega, N)\right)\frac{\sin\omega P_i}{\omega}d\omega\end{aligned} \tag{15}$$

The above expression can be numerically evaluated using series expansions to the desired accuracy [16].

Radius from ϵ to ∞: The characteristic function based approach is extended here to the scenario where the interfering radio nodes are assumed to be uniformly distributed in a concentric disc around the static receiver with inner radius ϵ and outer radius a. The probability density function of the node distribution with respect to the radius is now given by

$$f_r(r) = \frac{2r}{(a^2-\epsilon^2)}; \quad \epsilon \leq r \leq a \tag{16}$$

Let $X_{\epsilon,a}$ denote the sum interference power from the radios on this concentric disc.

$$X_{\epsilon,a} = \sum_{J_{\epsilon,a}} g(r_i) \tag{17}$$

Here, $J_{\epsilon,a}$ denotes the set of interfering nodes at distance r_i from the static receiver such that $\epsilon \leq r_i \leq a$. The characteristic function of $X_{\epsilon,a}$ is given by

$$\begin{aligned}&\phi_{X_{\epsilon,a}}(\omega, N) =\\ &\exp\left(N\pi\left(a^2-\epsilon^2\right)\left(\int_\epsilon^a \frac{2r}{(a^2-\epsilon^2)}e^{i\omega g(r)}dr - 1\right)\right)\end{aligned} \tag{18}$$

Using integration by parts and letting $a \rightarrow \infty$, the inner integral can be evaluated to get

$$\begin{aligned}&\phi_{X_{\epsilon,\infty}}(\omega, N) =\\ &\exp\left(N\pi\epsilon^2\left(1-e^{\frac{i\omega}{\epsilon^\alpha}}\right) - N\pi(-i\omega)^\beta\Gamma_{inc}\left(\frac{\omega}{\epsilon^\alpha}, 1-\beta\right)\right)\end{aligned} \tag{19}$$

Here, $(\Gamma_{inc}(.))$ is the incomplete Gamma function. The probability of outage for a threshold P_i can be calculated in a manner similar to Equation 15.

B. Gaussian Approximation with Correction Terms

An alternate way to approximate the sum interference at the static receiver, $X_{\epsilon,a}$, is to use a Gaussian approximation [1]. However, simulation results show that the distribution of $X_{\epsilon,a}$ is skewed to the left. This is due to the fact that interferers very close to the receiver terminal contribute a disproportionately large amount of interference. An Edgeworth expansion of the characteristic function (Equation 18) using higher order cumulants of $X_{\epsilon,a}$ is used to approximate the distribution of $X_{\epsilon,a}$ in [14]. The Edgeworth approximation amounts to a Gaussian distribution together with a skewness correction factor for $X_{\epsilon,a}$. This technique has also been analyzed in [15]. The probability density function for $X_{\epsilon,a}$ can be approximated by

$$p_{X_{\epsilon,a}}(x) \approx q_{(\hat{x})}(1 + t_1 + t_2) \tag{20}$$

Here, $\hat{x} = (x - mean(x))/\sqrt{variance(x)}$, $q(.)$ is the standard Normal density function with mean zero and variance one, and,

$$t_1 = \frac{k_3}{6}h_3(\hat{x}), \quad t_2 = \frac{k_4}{24}h_4(\hat{x}) + \frac{k_3^2}{72}h_6(\hat{x}). \tag{21}$$

$k_r = m_r m_2^{-r/2}$ for $r = 3, 4, ...$, where m_k is the k^{th} cumulant of $X_{\epsilon,a}$ and h_k are Hermite polynomials. The complementary cumulative distribution of $X_{\epsilon,a}$ can be approximated by

$$p_{X_{\epsilon,a}}(X_{\epsilon,a} > x) \approx 0.5\,erfc(\hat{x}) + q_{(\hat{x})}(1 + f_1 + f_2) \tag{22}$$

Here, *erfc*(.) is the complementary error function and

$$f_1 = \frac{k_3}{6}h_2(\hat{x}), \quad f_2 = \frac{k_4}{24}h_3(\hat{x}) + \frac{k_3^2}{72}h_5(\hat{x}). \tag{23}$$

This technique gives us an easier method to evaluate the distribution of the interference statistic than the previous method which uses numerical analysis. It is observed via simulations that the modified Gaussian distribution using Edgeworth expansion is a good approximation for the distribution of interference at the static receiver from interferers distributed in an annular disc around the static receiver when the inner radius ϵ is greater than or equal to 10.

IV. Outage Probability Distribution for Spectral Sharing Schemes

The outage probability distributions of the spectrum sharing schemes described in Section II are studied here for the scenario where no knowledge of the static radio system is available to the SS radios, the scenario where perfect knowledge of the locations and transmissions of static radios are available to the SS radios and the scenario where the SS radios scan their environment to obtain information about static radio transmissions.

A. No System Knowledge

SS radios are assumed to be distributed in a disc around the static receiver with the distance from the static receiver extending from zero to infinity. The SS radios have no information about the location of the static receiver and hence transmit as if there were no static receiver present. Interference is hence caused at the static receiver by transmissions from SS radios distributed over the entire disc with radius extending from zero to infinity.
The interference statistic at the static receiver is given by

$$X_{\infty} = \sum_{J_{0,\infty}} g\left(r_i\right). \tag{24}$$

The outage probability can now be calculated by using expression 15. The outage probability expression for the three spectrum sharing schemes considered in this paper are as follows.

1) Scheme-1: The outage probability for the overlay system is given by

$$\begin{aligned} p_{out}^{NB-IA}(P_i) &= Pr\left(P_a X_{\infty} > P_i\right) \\ &= \frac{2}{\pi}\int_0^{\infty} Re\left(\phi_{X_{\infty}}\left(\omega, N_n\right)\right)\frac{\sin\left(\omega P_i/P_a\right)}{\omega}d\omega. \end{aligned} \tag{25}$$

Here, N_n is the number of radios transmitting in the frequency used by the static receiver. Since an SS radio chooses a frequency from the available N_B frequencies with equal probability, $N_n = N/N_B$. It is shown in [14] that a subset of a Poisson process of radios chosen by an independent distribution on the radios, is also a Poisson process. Hence the N_n radios chosen by an independent uniform distribution on the available frequencies is also a Poisson process and the calculated characteristic function is directly applicable.

2) Scheme-2: The outage probability for the underlay system is given by

$$\begin{aligned} p_{out}^{WB}(P_i) &= Pr\left(\frac{P_a X_{\infty}}{N_B} > P_i\right) \\ &= \frac{2}{\pi}\int_0^{\infty} Re\left(\phi_{X_{\infty}}\left(\omega, N\right)\right)\frac{\sin\left(\omega N_B P_i/P_a\right)}{\omega}d\omega. \end{aligned} \tag{26}$$

3) Scheme-3: Since this scheme without knowledge of the static radio system is equivalent to Scheme-2, the expression for outage probability is the same as Equation 26

B. Perfect System Knowledge

The SS radios are assumed to have complete information about the location and transmissions of the static radio receiver and transmitter pair. An SS radio implementing IA is thus assumed to not transmit in a frequency band used by the static radio if its distance from the static receiver is such that its transmission could cause an outage at the static receiver. Let r_{min} denote this distance from the static receiver together with some safety margin. Hence no SS radio employing IA that exists within a radius of r_{min} from the static receiver transmits in the same frequency as the static receiver. This scheme thus precludes the possibility of outage being caused at the static receiver due to an individual SS radio transmission when a spectrum sharing scheme with IA is used. However, outage could still be caused at the static receiver due to the fact that the sum of the interference powers of the SS radios might be larger than the interference threshold (P_i) even if the interference caused by an individual SS radio is not. In other words, outage can be caused by the sum of interference power of SS radios uniformly distributed in a concentric disc with inner radius r_{min} and outer radius extending to infinity.
The interference statistic at the static receiver for a spectrum sharing scheme employing IA is given by

$$X_{r_{min},\infty} = \sum_{J_{r_{min},\infty}} g\left(r_i\right). \tag{27}$$

1) Scheme-1: The outage probability for the overlay system is given by

$$\begin{aligned} p_{out}^{NB-IA}(P_i) &= Pr\left(P_a X_{r_{min},\infty} > P_i\right) \\ &= \frac{2}{\pi}\int_0^{\infty} Re\left(\phi_{X_{\epsilon,\infty}}\left(\omega, N_n\right)\right)\frac{\sin\left(\omega P_i/P_a\right)}{\omega}d\omega. \end{aligned} \tag{28}$$

Here, $\phi_{X_{\epsilon,\infty}}(.)$ is the characteristic function of $X_{\epsilon,\infty}$ defined in Equation 19 and $\epsilon = r_{min}$.

2) Scheme-2: The outage probability expression for the underlay system is the same as Equation 26, since this scheme does not require knowledge of the static radio system.

3) Scheme-3: The outage probability for the overlay system with IA is given by

$$\begin{aligned} p_{out}^{WB}(P_i) &= Pr\left(\frac{P_a X_{r_{min},\infty}}{N_B} > P_i\right) \\ &= \frac{2}{\pi}\int_0^{\infty} Re\left(\phi_{X_{\epsilon,\infty}}\left(\omega, N\right)\right)\frac{\sin\left(\omega N_B P_i/P_a\right)}{\omega}d\omega. \end{aligned} \tag{29}$$

When $r_{min} \geq 10$, the Gaussian approximation described in Section III-B can be used to approximate the outage probability distributions instead of numerical analysis. Expression 22 can be used to determine the outage probability for a given interference threshold. If N is the number of interfering radios per unit area and if $g\left(r\right)$ is as defined in Equation 7, the k^{th} cumulant of $X_{r_{min},\infty}$ is given by

$$m_k = \frac{2N\pi}{(k\alpha - 2)\left(r_{min}^{(k\alpha-2)}\right)}. \tag{30}$$

C. Imperfect System Knowledge

The previous section assumed that the SS radio schemes employing IA have perfect knowledge of the locations of the static radio receiver and transmitter relative to their own locations. However, this requires a large overhead in the network. A more practical approach is one in which the SS radios scan their environment for static radio transmissions and do not transmit in the frequency band in which they sense some transmission. In this scenario, interference is caused to the static radio system by transmissions from SS radio nodes which are hidden from the static radio transmitter (are outside the range in which the SS radios can sense transmissions from the static transmitter). In this section, expressions are derived for the outage probability of the static receiver that incorporate the effects of imperfect sensing due to the presence of hidden SS radio nodes. In the analysis, the radios are assumed to be able to perfectly sense the static radio transmission when inside the sensing range. The effect of imperfect sensing will be included in our future work.

Let r_s be the distance between the static transmitter and static receiver. Let r_{as} be the range from the static transmitter within which the SS radio can identify static radio transmissions. Let θ be the angle of the line joining the SS radio transmitter and the static receiver with respect to the line joining the static transmitter and receiver. These variables are illustrated in Figure 1. Consider a disc of radius a. Then the area outside the sensing region is given by $\pi\left(a^2 - r_{as}^2\right)$. The probability density function of the node distribution with respect to radius r and θ is given by

$$f_{r,\theta}\left(r,\theta\right) = \frac{r}{\pi\left(a^2 - r_{as}^2\right)}; \quad \begin{array}{c} r_{min}\left(\theta\right) \leq r \leq a \\ 0 \leq \theta \leq 2\pi \\ . \end{array} \tag{31}$$

Here, $r_{min}\left(\theta\right)$ is the minimum distance of the agile transmitter from the static receiver such that the static transmitter is hidden from it. It is a function of θ and is given by (Figure 1)

$$r_{min}(\theta) = r_s \cos\theta + r_{as} \sin\left(\cos^{-1}\left(\frac{r_s \sin\theta}{r_{as}}\right)\right). \tag{32}$$

The interference statistic at the static receiver for a spectrum sharing scheme employing IA in the absence of perfect system knowledge is given by

$$X_{\epsilon,a}^{h} = \sum_{J_{r_{min}(\theta),a}^{imp}} g\left(r_i\right). \tag{33}$$

Here, $J_{r_{min}(\theta),a}^{imp}$ is the set of SS radios that are outside the sensing region of the static transmitter, i.e., r_i satisfies $r_{min}\left(\theta\right) \leq r_i \leq a$ and θ satisfies $0 \leq \theta \leq 2\pi$.

Letting $a \rightarrow \infty$, the characteristic function of $X_{\epsilon,\infty}^{h}$ can be evaluated to get,

$$\begin{aligned} \phi_{X_{\epsilon,\infty}^{h}}\left(\omega, N\right) &= e^{\left(N\pi r_{as}^2 - \frac{N}{2}\int_0^{2\pi} r_{min}^2(\theta) e^{\frac{i\omega}{r_{min}^{\alpha}(\theta)}} d\theta\right)} \\ &\times e^{\left(-\frac{N(-i\omega)^{\beta}}{2}\int_0^{2\pi} \Gamma_{inc}\left(\frac{\omega}{r_{min}^{\alpha}(\theta)}, 1-\beta\right) d\theta\right)}. \end{aligned} \tag{34}$$

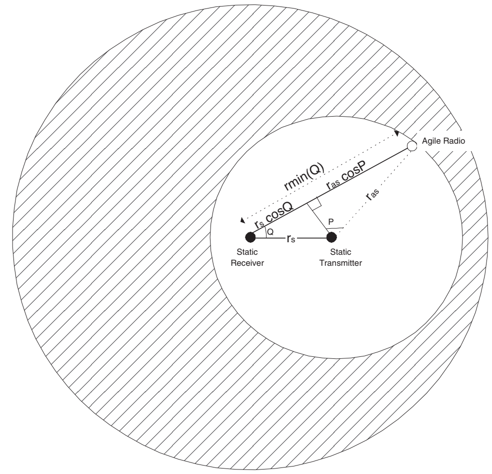

Fig. 1. The shaded region is the hidden node region around a static transmitter. Q is used to represent θ in the figure.

1) Scheme-1: The outage probability for the overlay system with imperfect system knowledge is given by

$$\begin{aligned} p_{out}^{NB-IA}(P_i) &= Pr\left(P_a X_{\epsilon,\infty}^{h} > P_i\right) \\ &= \frac{2}{\pi}\int_0^{\infty} Re\left(\phi_{X_{\epsilon,\infty}^{h}}\left(\omega, N_n\right)\right)\frac{\sin\left(\omega P_i/P_a\right)}{\omega} d\omega. \end{aligned} \tag{35}$$

Here, $\phi_{X_{\epsilon,\infty}^{h}}\left(.\right)$ is the characteristic function of $X_{\epsilon,\infty}^{h}$ and is defined in Equation 34.

2) Scheme-2: The outage probability expression for the underlay system is the same as Equation 26, since the scheme does not require knowledge of the static radios.

3) Scheme-3: The outage probability for the overlay system with IA and imperfect system knowledge is given by

$$p_{out}^{WB}(P_i) = Pr\left(\frac{P_a X_{\epsilon,\infty}^{h}}{N_B} > P_i\right). \tag{36}$$

When $min\left(r_{min}(\theta)\right) \geq 10$, the Gaussian approximation described in Section III-B can be used to approximate the outage probability distributions instead of numerical analysis. Expression 22 can be used to determine the outage probability for a given interference threshold. The cumulants for $X_{\epsilon,\infty}^{h}$ can be calculated using numerical integration.

V. Outage Probability Analysis and Comparison of Spectral Sharing Schemes

Some example scenarios are used here to analyze and compare the outage caused at the static radio receiver due to the different spectrum sharing schemes. In all the scenarios discussed, the transmit power of the SS radio is taken to be 10 mwatts and the power loss at a distance of $1m$ is assumed to be -40dB (Hence, $P_a = -60$dBw). This attenuation is typical of indoor channels. The interference statistics can be appropriately scaled if this is not the case. This scaling does not influence the comparative performance results of different spectrum sharing schemes. The path loss exponent, α, is 3. Unless otherwise mentioned, the average number of SS radio nodes per unit area (in m^2), N, is taken to be 1.

A. No System Knowledge

The expressions for outage probability in Section IV-A are plotted in Figure 2 for different interference thresholds (Tolerable Interference Power). The expressions were evaluated using numerical integration. The radius of the disc in which the interferers are distributed extends from 0 to ∞. It is observed that for a given interference threshold the outage caused by the NB overlay scheme is less than the WB underlay scheme. However, allowing the radius of the disc to extend from zero is artificial, since an SS radio and a static receiver cannot share the same location. Due to this physical limitation, it is justified to assume that SS radios do not exist in a distance closer than 0.1m($= 10$cm) from the static receiver. The outage probability of the static receiver for this scenario where the radius of the disc in which the interferers are distributed ranges from 0.1m to ∞ is plotted in Figure 3 and Figure 4. It is seen that when low outage probabilities are required, the interference power required to be tolerated at the static receiver is smaller in the case of the underlay scheme. However, when high outage probabilities can be tolerated at the static receiver, the overlay scheme is preferred. It is expected that such a crossover would also show up when the interferers are distributed in a disc that extends from 0 to ∞. However, the crossover point occurs at very large interference thresholds. From the figures, it is also seen that the difference in the slopes of the curves is larger for larger bandwidths, showing that the performance improvement of the underlay system over the overlay system at low outage probabilities, increases with an increase in bandwidth. It is to be noted that the interference caused at the static receiver is very large when no information about the static receiver is available to the SS radios and the radios do not implement any form of interference avoidance.

B. Perfect System Knowledge

For ease of analysis, example scenarios are chosen such that the Gaussian approximation method described in Section III-B can be used. The characteristic function based numerical approach can be used to evaluate the performance for other scenarios. However, this approach is computationally intensive. We are currently working on analytical performance bounds that are not as computationally involved. Preliminary results show performance trends similar to those presented here.

The static transmitter is assumed to be at distance of 5m from the static receiver and is assumed to transmit at a power of 100 mwatts. An SS radio implementing IA is assumed to not transmit in the same frequency as the static transmitter if its distance from the static receiver is less than 20m ($r_{min} = 20$). This ensures that up to a signal to interference ratio (SIR) threshold of 30dB, no outage is caused at the static receiver due to a NB transmission from a single SS radio. To ensure a fair comparison, WB SS radios implementing IA which are within the same distance from a static receiver are assumed to not transmit in the same frequency as the static transmitter. Since $r_{min} > 10$, the Gaussian approximation method is used to evaluate the outage probabilities for this scenario. Outage probabilities for Scheme-1 and Scheme-3 with $r_{min} = 20$m are plotted in Figure 5 and Figure 6 for $N_B = 8$ and $N_B = 64$ respectively. Outage probabilities with $r_{min} = 50$m are plotted in Figure 7 and Figure 8. The outage probability for Scheme-2 would be similar to the plots obtained in Figures 2 - 4, since the presence or absence of information about the static radio system does not affect the transmission of a spreading-based underlay system that does not implement IA. By comparing Figures 5 - 8 with Figures 2- 4, it can be noticed that implementing IA dramatically reduces the outage probabilities at the static receiver. Hence some sort of IA must be implemented by the underlay scheme to make it comparable to the IA-based overlay scheme. However, it can be observed from Figures 5 - 8 that when implementing IA the spreading-based underlay scheme (Scheme-3) guarantees smaller outage probabilities than the IA-based overlay scheme for a given interference threshold, excepting for a small cross-over region. It can also be noticed that the interference reduction to the static receiver while using Scheme-3 as compared to Scheme-1 is more pronounced when a larger bandwidth, N_B, is available to the SS radio system.

C. Imperfect System Knowledge

For ease of analysis, example scenarios are chosen such that the Gaussian approximation method can be used. The static transmitter is assumed to be at distance of 5m from the static receiver ($r_s = 5$) and is assumed to transmit at a power of 100 mwatts. The radius of the sensing region around the static transmitter is assumed to be 15m ($r_{as} = 15$). The received power at this distance is around 85dB less than the transmit power due to path loss. The outage probability distribution for this scenario is computed using the Gaussian approximation method described in Section III-B since $min(r_{min}(\theta)) = r_{as} - r_s = 10$m. The cumulants of the interference statistic, $X^h_{\epsilon,\infty}$, at the static receiver is computed from the density function given in Equation 31 using numerical integration. Outage probability distributions for $N_B = 8$ and $N_B = 64$ are plotted in Figure 9 and Figure 10, respectively. The figures also show the outage probabilities for the scenario where the SS radios have perfect knowledge of the static radio system and $r_{min} = 20$m. Greater interference is seen to be caused at the static receiver when imperfect information due to the hidden node problem as opposed to perfect information about the static receiver is available to the SS radio nodes. This is due to presence of SS radios whose distance from the static receiver is less than r_{min} and which are hidden from the static transmitter, causing larger interference at the static receiver.

Comparing Scheme-1 and Scheme-3 in Figure 9 and Figure 10, it is seen that the IA-based overlay scheme causes more interference at the static receiver. It is also seen that the increase in interference caused at the static receiver in the absence of perfect information is greater for the IA-based overlay scheme as compared to the IA-based underlay scheme. This is intuitive since in the absence of perfect information due to the hidden node problem, some nodes that are close to the static receiver interfere with the static radio transmission. In this scenario, it is beneficial if the transmit power of these SS radio nodes is distributed over all frequencies (as is the case for an underlay scheme) instead of being concentrated in the transmission frequency of the static radio (as is the case for an overlay scheme). These results are re-iterated in Figure 11, Figure 12 and Figure 13 which show the outage probabilities at the static receiver for different radio densities. It is observed that in general, the density of SS radios that can be supported using an IA-based underlay approach is greater than the density that can be supported using an IA-based overlay scheme for a given outage probability. Also, the density of SS radios that can be supported by both schemes when perfect information is available at the SS radio nodes is greater than the density that can be supported when perfect information is unavailable. However, the reduction in the density of nodes that can be supported in the absence of perfect information is greater in the case of the IA-based overlay system as compared to the IA-based underlay system. Additionally, it can be observed from the figures that the margin of density reduction, due to the absence of perfect information, of the underlay system over the overlay system increases with increased available bandwidth N_B.

VI. Conclusions

Three spectrum sharing schemes - spreading-based underlay, IA-based overlay and spreading-based underlay that implements IA, were compared in this paper. The comparison was based on the interference caused by these schemes on a static radio network. A characteristic function based numerical approach and a Gaussian approximation based approach were used to model the interference statistics at a static receiver. Outage probabilities at the static receiver with the three different spectrum sharing schemes were derived and analyzed. Example scenarios were used to illustrate the performance trends.

It is shown that IA techniques dramatically reduce the interference seen at the static radios. Hence spectral underlay schemes need to incorporate IA to be comparable to IA-based spectral overlay schemes. However, our analysis also shows that in general, spectrum underlay schemes that employ IA result in lower outage probabilities for the static radio system as compared to IA-based spectrum overlay schemes. The benefits provided by the IA-based underlay scheme over the IA-based spectral overlay scheme is shown to be more pronounced as the transmission bandwidth available to the SS radio system is increased. This motivates the use of UWB-based spectrum underlay techniques for spectrum sharing. In addition, the IA-based underlay scheme is less affected by the absence of perfect information about the static radio system required for IA. This provides a larger flexibility in the practical deployment of IA-based underlay schemes than IA-based overlay schemes for spectrum sharing.

In the future, we propose to extend our analysis to include the effects of shadowing, fading and imperfect sensing. Also, an ideal spectrum sharing scheme for an agile network would be one which minimizes interference to the legacy static radio system while maximizing the capacity of the SS radio network. Hence we also intend to investigate outage probability distributions for the SS radio system.

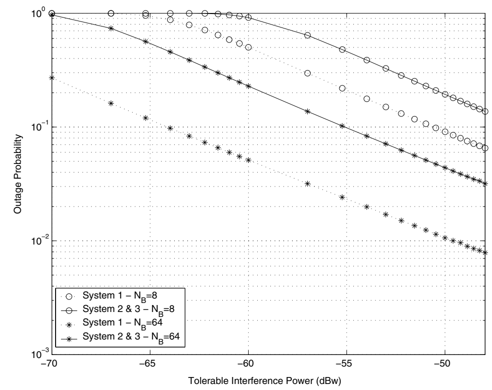

Fig. 2. Outage probability vs interference threshold when the SS radios have no information about the static radio system. Interferers are distributed in a disc with radius extending from 0 to ∞

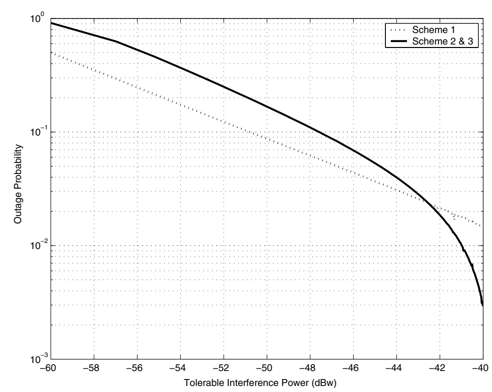

Fig. 3. Outage probability vs interference threshold when the SS radios have no information about the static radio system. Interferers are distributed in a disc with radius extending from 0.1m to ∞, $N_B = 8$

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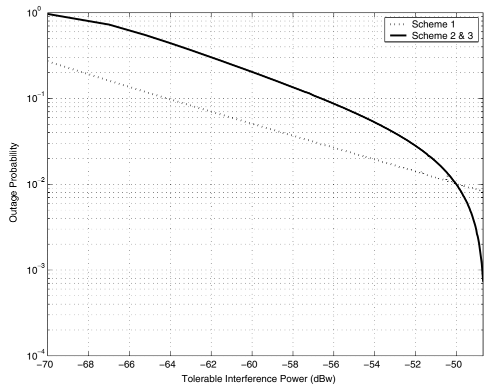

Fig. 4. Outage probability vs interference threshold when the SS radios have no information about the static radio system. Interferers are distributed in a disc with radius extending from 0.1m to ∞, $N_B = 64$

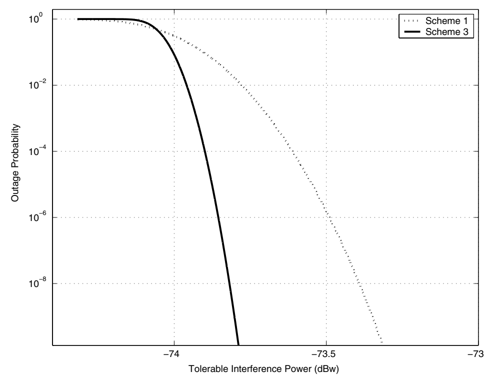

Fig. 5. Outage probability vs interference threshold when the SS radios have perfect information about the static radio system. Interferers are distributed in a disc with radius extending from $20m$ to ∞, $N_B = 8$

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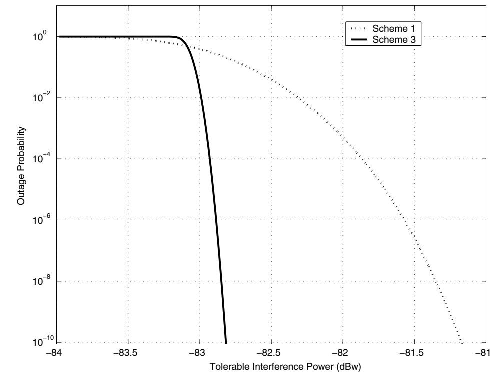

Fig. 6. Outage probability vs interference threshold when the SS radios have perfect information about the static radio system. Interferers are distributed in a disc with radius extending from $20m$ to ∞, $N_B = 64$

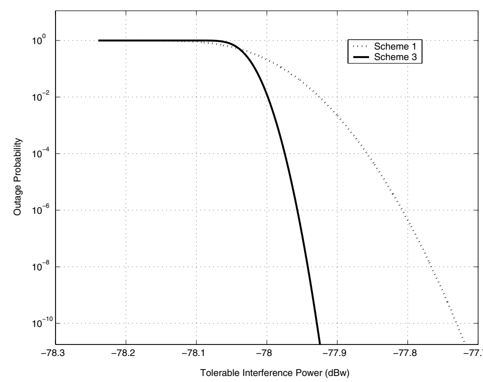

Fig. 7. Outage probability vs interference threshold when the SS radios have perfect information about the static radio system. Interferers are distributed in a disc with radius extending from $50m$ to ∞, $N_B = 8$

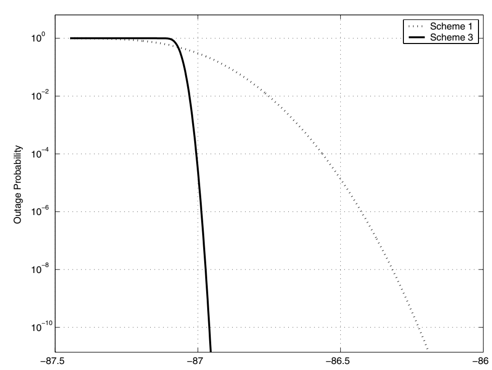

Fig. 8. Outage probability vs interference threshold when the SS radios have perfect information about the static radio system. Interferers are distributed in a disc with radius extending from $50m$ to ∞, $N_B = 64$

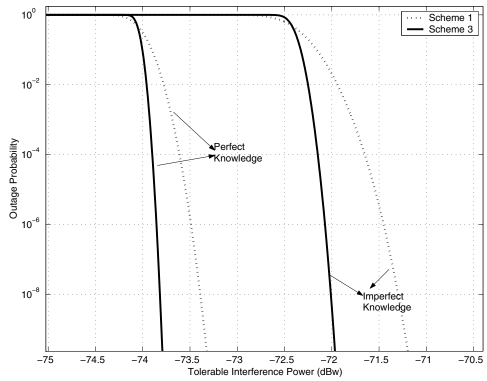

Fig. 9. Outage probability vs interference threshold when the SS radios have perfect and imperfect information about the static radio system. For the imperfect information scenario, the distance between the static transmitter and receiver, $r_s = 5m$ and the sensing radius, $r_{as} = 15m$. For the perfect information scenario, the radius around the static receiver in which no interferers are present is $20m$. $N_B = 8$.

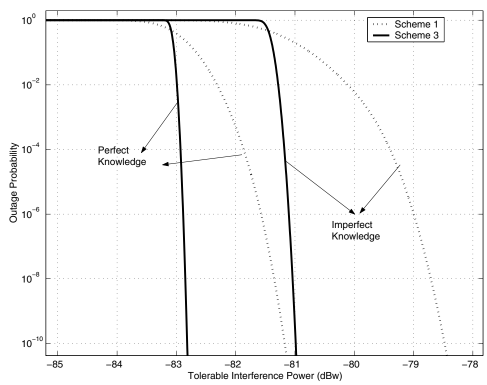

Fig. 10. Outage probability vs interference threshold when the SS radios have perfect and imperfect information about the static radio system. For the imperfect information scenario, the distance between the static transmitter and receiver, $r_s = 5m$ and the sensing radius, $r_{as} = 15m$. For the perfect information scenario, the radius around the static receiver in which no interferers are present is $20m$. $N_B = 64$.

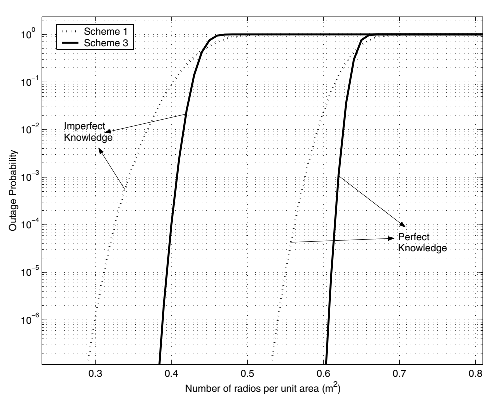

Fig. 11. Outage probability vs the density of SS radio nodes. SIR threshold, $\gamma = 5dB$. $N_B = 8$.

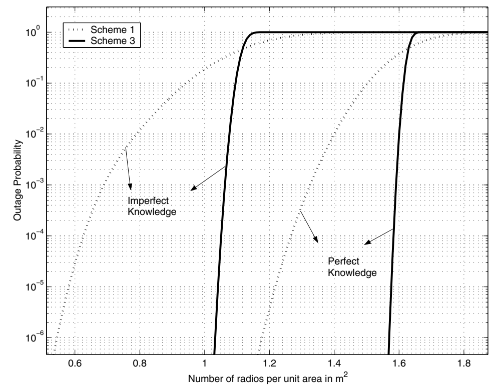

Fig. 12. Outage probability vs the density of SS radio nodes. SIR threshold, $\gamma = 10dB$. $N_B = 64$.

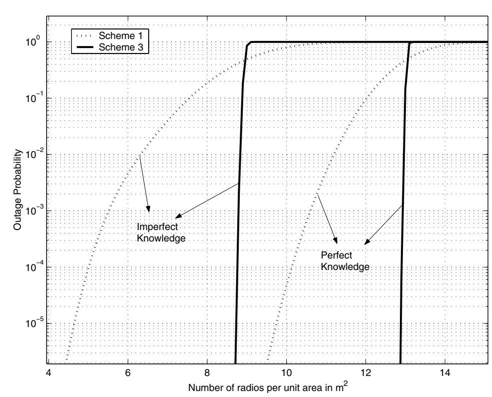

Fig. 13. Outage probability vs the density of SS radio nodes. SIR threshold, $\gamma = 10dB$. $N_B = 512$.